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There is no attempt at accuracy in the diagrams. These are merely intended to give a general idea of the relation of height to ebb and flow. The vertical distances are of course much exaggerated.

Haverford College, November, 1899.

NOTE ON NETTO'S THEORY OF SUBSTITUTIONS.

By G. A. Miller.

In finding a function that belongs to a given group, Netto employs a method which is apt to mislead the student. Since some other authors express themselves somewhat indefinitely* in regard to this fundamental problem, it may be desirable to give several elementary illustrations which will exhibit the points in question. We shall first show how the given method may lead to incorrect results. Employing the same Galois function ($\phi_1 = x_1 + ix_2 - x_3 - ix_4$) as Netto employs† we proceed to find a function belonging to the group $G_2 = 1, (x_1x_2)(x_3x_4)$ by the first method which he employs to find a function belonging to the group

$$G_8 = 1,\ (x_1x_2)(x_3x_4),\ (x_1x_3)(x_2x_4),\ (x_1x_4)(x_2x_3),\ (x_1x_3),\ (x_2x_4),\ (x_1x_2x_3x_4),\ (x_1x_4x_3x_2).$$

Transforming ϕ_1 by the substitutions of G_2 we have:

$$\psi_1 = \phi_1\phi_2 = (x_1 + ix_2 - x_3 - ix_4)(x_2 + ix_1 - x_4 - ix_3) = i(x_1 - x_3)^2 + i(x_2 - x_4)^2.$$

If we transform ϕ_1 by the substitutions of $G'_2 = 1, (x_2x_4)$ we obtain:

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* Vogt, *Résolution algébrique des équations*, 1895, p. 23.

† Netto, *Theory of Substitutions*, translated by Cole, 1892, p. 30.

$\psi_2 = -i\phi_1\phi_2 = (x_1 + ix_2 - x_3 - ix_4)(x_1 + ix_4 - x_3 - ix_2) = (x_1 - x_3)^2 + (x_2 - x_4)^2$. Finally, we transform ϕ_1 by the substitutions of $G''_2 = 1, (x_1x_3)(x_2x_4)$ and obtain: $\psi_3 = -\phi_1^2 = (x_1 + ix_2 - x_3 - ix_4)(x_3 + ix_4 - x_1 - ix_2) = -(x_1 + ix_2 - x_3 - ix_4)^2$.

It is evident that each of the functions ψ_1, ψ_2, admits all the substitutions of G_8. They admit no other substitution in these elements since G_8 is not contained in any group in these elements besides the symmetric group,* and since they clearly do not admit the transposition (x_1x_4). Hence each one of these two functions belongs to G_8 and not to the given groups of order two. This shows that the method which Netto employed to construct a function belonging to G_8 cannot be employed (if the given Galois function is used as the fundamental function) to construct a function belonging to either of the two given groups of order two. It may readily be verified that similar considerations in regard to the other two non-selfconjugate subgroups of order two that are contained in G_8 lead to the functions $-\phi_1\phi_2$ and $i\phi_1\phi_2$ respectively, each of which belongs to G_8.

With respect to G''_2 the matter is entirely different. ψ_3 actually belongs to this group since it clearly admits its substitutions and no others. These results follow very easily from the fact that the cyclical subgroup of order four contained in G_8 transforms ϕ_1 into itself multiplied by the four fourth roots of unity, while the remaining four substitutions of G_8 transform it into ψ_2 multiplied by the same four roots. Hence we observe that the given method together with the given fundamental Galois function can be employed in only one out of the five cases to construct a function belonging to a group of order two contained in G_8. From this it follows directly that this method can be employed in only one of the three cases to construct a function belonging to a group of order four that is contained in G_8, viz., when this group of order four is cyclical.†

In order to see clearly where the difficulty lies it may be well to observe that if a substitution transforms a function into itself multiplied by a constant it is necessary that the absolute value of this constant be unity; for from the equation $s^{-1}Fs = \rho F$ we obtain $s^{-n}Fs^n = \rho^n F = F$, where n is the order of s. If we transform any linear function of n independent elements by any substitution in these elements, the result is evidently equivalent to transforming its coefficients in the reverse order. Hence *s cannot transform a linear Galois function into itself multiplied by a constant (ρ), which is not unity, unless all*

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* Cf. Cayley, *Quarterly Journal of Mathematics*, vol. 25, 1891, p. 77.

† Vogt, *l. c.*

the coefficients of these elements can be divided into distinct sets containing a multiple of n numbers and having a common absolute value. In particular, any linear Galois function in which the absolute value of one coefficient is larger than the absolute value of any one of the others cannot be transformed (by means of a substitution) into itself multiplied by a constant.

If we employ such a Galois function (f_1) as a fundamental function, the method employed by Netto will clearly always lead to a function (F) belonging to the group: for if $f_1, f_2, \ldots, f_\lambda$ are the functions obtained by transforming f_1 by the substitutions of this group (T), then $F = f_1 f_2 f_3 \ldots f_\lambda$ is evidently transformed into itself by T. That it is not transformed into itself by any other substitution follows from the fact that such a substitution would transform the given linear factors of F into a new set none of which could be obtained by multiplying a factor of F by a constant. Since an expression can be resolved into linear factors in only one way,* the product of this second set of factors could not be equal to F. If we assume that a linear Galois function contains a constant term which is not zero, it is well known† that this also may always be employed to construct a function (according to the given method) that belongs to any given group.

CORNELL UNIVERSITY, OCTOBER, 1899.

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* Cf. Weber, *Algebra*, Vol. 1, 1898, p. 74.

† Cf. Vogt, *Résolution algébrique des équations*, 1895, p. 23.